

Stochastic resonance as a filter for signal detection from multi signal inputs

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Abstract. We undertake a detailed numerical study of the phenomenon of stochastic resonance with multisignal inputs. A bistable cubic map is used as the model and we show that it combines the features of a bistable system and a threshold system. A study of stochastic resonance in these two setups reveal some fundamental differences between the two mechanisms with respect to amplification of a composite input signal. As a practically relevant result, we show that the phenomenon of stochastic resonance can be used as a *filter* for the detection/transmission of the component frequencies in a composite signal.

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1. Introduction

Stochastic Resonance(SR) refers to the situation wherein the response of a nonlinear system to a weak input signal can be significantly increased with appropriate tuning of the noise intensity[1,2]. When a subthreshold signal $I(t)$ is input to a nonlinear system g together with a noise $\zeta(t)$, if the filtered output $O(t) \equiv g(I(t) + \zeta(t))$ shows enhanced response that contains the information content of $I(t)$, then SR is said to be realised in the system. The mechanism first used by Benzi, Nicolis etc [3,4] to explain natural phenomena is now being used for a large variety of interesting applications like modelling biological and ecological systems [5], lossless communication purposes etc [6]. Apart from these, it has opened up a vista of many related resonance phenomena [7] which are equally challenging from the point of view of intense research. In this work, we try to capture some of these, using a simple model system, namely, a two parameter bimodal cubic map.

In the early stages of the development of SR, most of the studies were done using a dynamical set up with bistability, modelled by a double well potential. Here SR is realised due to the shuttling between the two stable states at the frequency of the subthreshold signal with the help of noise. Thus if the potential is

$$V(X) = -aX^2/2 + bX^4/4 \quad (1)$$

in the presence of a signal and noise, the dynamics can be modelled by an overdamped oscillator

$$\dot{X} = aX - bX^3 + Z\sin\omega t + E\zeta(t) \quad (2)$$

If C_{th} is the threshold at which deterministic switching (with noise amplitude $E = 0$) is possible, then well to well switching due to SR occurs when

$$-C_{th} > (Z\sin\omega t + E\zeta(t)) > C_{th} \quad (3)$$

that is, twice in one period of the signal $T = 2\pi/\omega$.

The characterisation of SR in this case is most commonly done by computing the signal to noise ratio (SNR) from the power spectrum of the output as

$$SNR = 10\log_{10}(S/N)dB \quad (4)$$

where N is the average background noise around the signal S . If SR occurs in the system, then the SNR goes through a maximum giving a bell shaped curve as E is tuned.

SR has also been observed in systems with a single stable state with an escape scenario. These *threshold* systems, in the simplest case, can be modelled by a piecewise linear system or step function

$$g(u) = -1 \quad u < C_{th}$$

$$= +1 \quad u > C_{th}$$

The escape with the help of noise is followed by resetting to the monostable state. In this case, a quantitative characterisation is possible directly from the output, but only in terms of probabilities. If t_n are the escape times, the inter spike interval (ISI) is defined as $T_n = t_{n+1} - t_n$ and $m(T_n)$ is the number of times the same T_n occurs. For SR to be realised in the system, the probability $p_n = m(T_n)/N$ (N is the total number of escapes) has to be maximum corresponding to the signal period T at an optimum noise amplitude.

There are situations where SR is to be optimised by adapting to or designing the dynamics of the system. This is especially relevant in natural systems or electronic circuits where the noise is mostly from the environmental background and therefore not viable to fine tuning. Similarly, depending on the context or application, the nature of the signal can also be different, such as, periodic, aperiodic, digital, composite etc. The classical SR deals with the detection of a single subthreshold signal immersed in noise. However, in many practical situations, a composite signal consisting of two or more harmonic components in the presence of background noise is encountered, as for example, in biological systems for the study of planktons and human visual cortex [8], in laser physics [9] and in acoustics [10]. Moreover, two frequency signals are commonly used in multichannel optical communication systems based on wave length division multiplexing (WDM) [11]. But very few studies of SR have been carried out involving such *bichromatic* signals to date [12-15], and each of them pertaining to some specific dynamical set ups. This motivates us to undertake a detailed numerical analysis of SR with such signals using a simple model, a two parameter cubic map. It is a discretised version of the overdamped bistable oscillator, but with the added feature of an inherent escape mode also. Hence it can function in both set ups, bistable and threshold, as a stochastic resonator.

Our analysis brings out some generic features (and also a few interesting differences) of both these mechanisms of SR with respect to multisignal inputs. An especially important result that has emerged from our studies is that SR can, in principle, be used as a *filter* to detect the fundamental frequencies in a composite signal by tuning the noise amplitude. Our paper is organised as follows: In §2, we introduce the cubic map and discuss some theoretical aspects of SR with multiple signals. In §3, we study SR in the system numerically with a composite signal treating it as a bistable system. §4 considers the cubic map as a threshold system having properties different from that of a bistable system. Results and discussions are given in §5.

2. The model

The two parameter cubic map is given by

$$X_{n+1} = f(X_n) = b + aX_n - X_n^3 \quad (5)$$

The system has been studied in great detail both analytically and numerically and has been shown to possess a rich variety of dynamical properties including bistability [16].

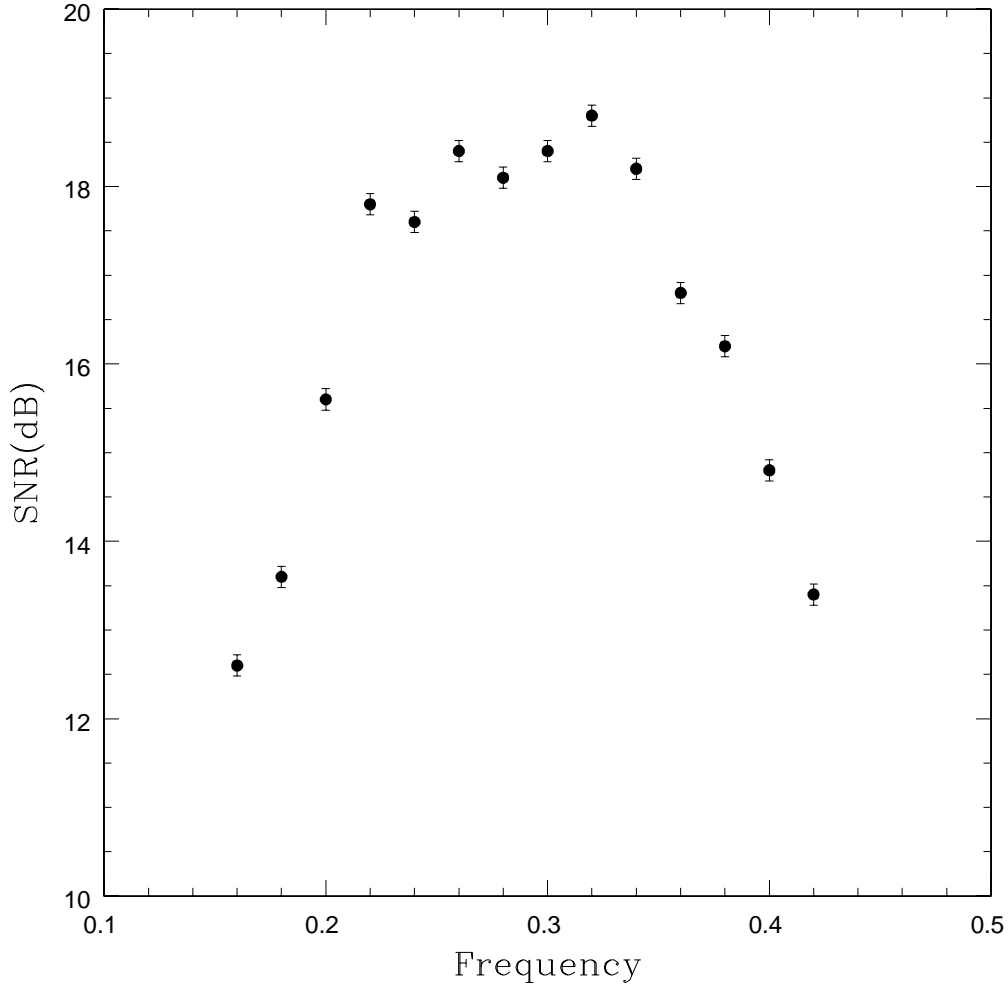


Figure 1. Variation of SNR with frequency for the bistable cubic map in the chaotic regime ($a = 2.4$) with the noise amplitude $E = 0$. Note that a subthreshold signal ($Z = 0.16$) can be detected for an intermediate range of frequencies, without any external noise.

In particular, if a_1 is the value of the parameter at which $f'(X_i, a_1, b) = 1$, then for $a > a_1$, there is a window in b , where bistability is observed. The bistable attractors are clearly separated with $X > 0$ being the basin of one and $X < 0$ that of the other. For example, for $a = 1.4$, $b = [-0.1, 0.1]$, two attractors of period 1 coexist. As the value of a is increased, the periodicity of the bistable attractors keeps on doubling while the width of the window around b decrease correspondingly. Finally, for $a = 2.4$, two chaotic attractors co-exist in a very narrow window around b .

The system when driven by a gaussian noise and a periodic signal becomes

$$X_{n+1} = b + aX_n - X_n^3 + E\zeta(t) + ZF(t) \quad (6)$$

where we choose $\zeta(t)$ to be a gaussian noise with zero mean and $F(t)$ is the periodic signal. The amplitude of the noise and the signal can be varied by tuning E and Z respectively. It can be shown that in the regime of chaotic bistable attractors ($a = 2.4, b = 0.01$), a subthreshold input signal can be detected using the inherent chaos in the system without any external noise ($E=0$). Taking the signal $F(t) = Z\sin(2\pi\nu t)$, with $Z = 0.16$, the system shows SR type behavior for an optimum range of frequencies as shown in Fig.1, where the output SNR is plotted as a function of the frequency ν . It implies that a subthreshold signal can be detected by passing through a bistable system making use of the inherent chaos in it without the help of any external noise. This phenomenon is known as *deterministic resonance*. In the regime of periodic bistable attractors ($a = 1.4, b = 0.01$) with a single subthreshold signal, the system shows conventional SR as well as chaotic resonance (CR), and using this model, we have recently reported some new results including enhancement of SNR via coupling [17].

For the remaining part of the paper we consider the signal $F(t)$ to be a combination of at least 2 frequencies, ν_1 and ν_2 . A typical input signal is shown in Fig.2(c) which is a superposition of two fundamental frequencies $\nu_1 = 0.125$ and $\nu_2 = 0.033$ shown in Fig.2(a) and 2(b) respectively. Before we go into the numerical studies, we consider some theoretical results for multisignal inputs. An analytical description of SR usually considers the model of an overdamped bistable oscillator in a double well potential, driven by white noise $\zeta(t)$ and a periodic signal $F(t) = Z\sin(2\pi\nu t)$. An expression for the SNR can be derived using some approximate theories, the most popular being the Linear Response Theory (LRT) [18-21].

According to this theory, the response of a nonlinear stochastic system $X(t)$ to a weak external force $F(t)$ in the asymptotic limit of large times is determined by the integral relation [18]

$$X(t) = \langle X_0 \rangle + \int_{-\infty}^{\infty} R(t - \tau) F(\tau) d\tau \quad (7)$$

where $\langle X_0 \rangle$ is the mean value of the state variable for $F(t) = 0$. Without lack of generality, one can set $\langle X_0 \rangle = 0$. The function $R(t)$ in (7) is called the response function. For a harmonic signal, the system response can be expressed through the function $R(\omega)$ which is the Fourier transform of the response function:

$$X(t) = Z|R(\omega)|\sin(2\pi\nu t + \psi) \quad (8)$$

where ψ is a phase shift.

The LRT can be naturally extended to the case of multifrequency signals. Let the signal $F(t)$ be a composite signal of the form:

$$F(t) = Z \sum_{k=1}^n \sin(2\pi\nu_k t) \quad (9)$$

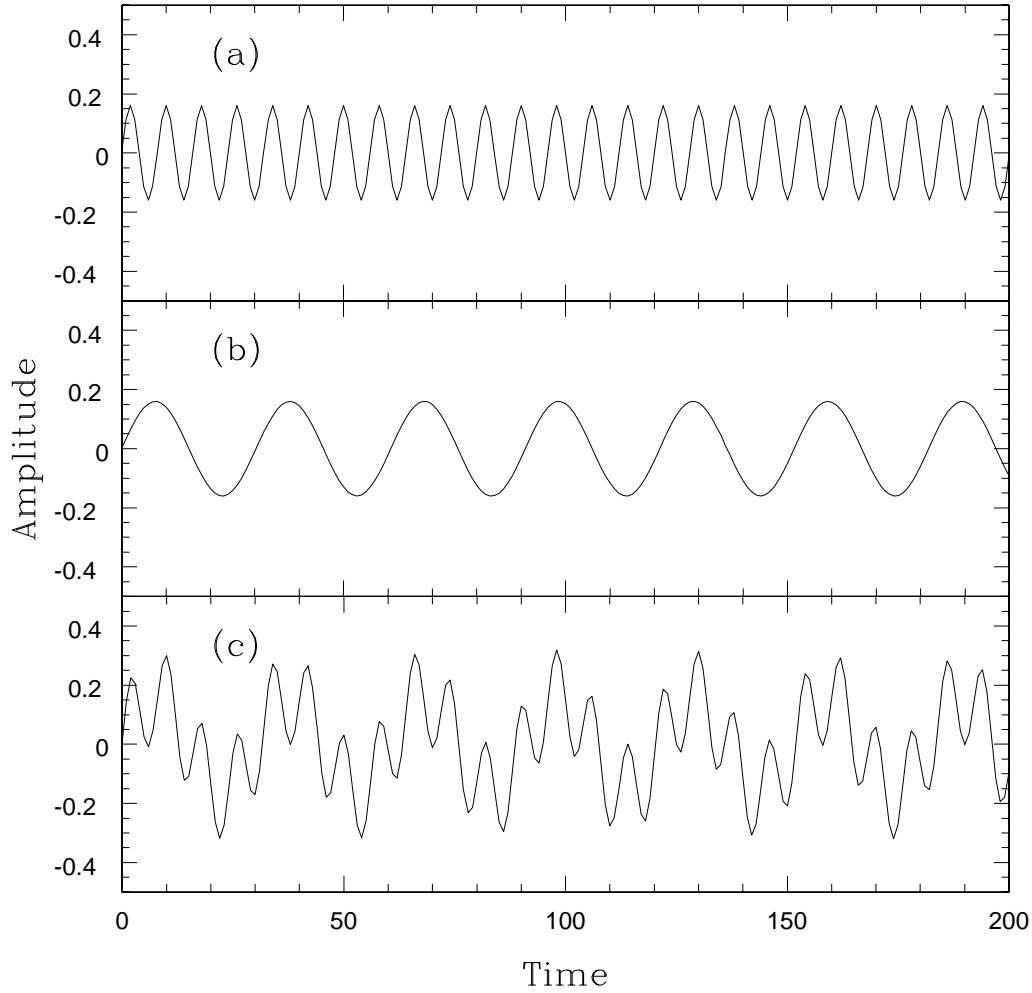


Figure 2. Time variation of a signal with frequency (a) $\nu_1 = 0.125$ and (b) $\nu_2 = 0.033$, with amplitude $Z = 0.16$ in both cases. When these signals are superposed, the resulting signal is shown in fig.(c), which is in accordance with the linear superposition principle.

where ν_k are the frequencies of the discrete spectral components with the same amplitude Z . Then according to LRT, the system response can be shown to be [21]

$$X(t) = Z \sum_{k=1}^n |R(\omega_k)| \sin(2\pi\nu_k t + \psi_k) \quad (10)$$

which contains the same spectral components at the input, but with different amplitudes and phases. We now investigate this numerically in more detail using system (5), both as a bistable system and a threshold system.

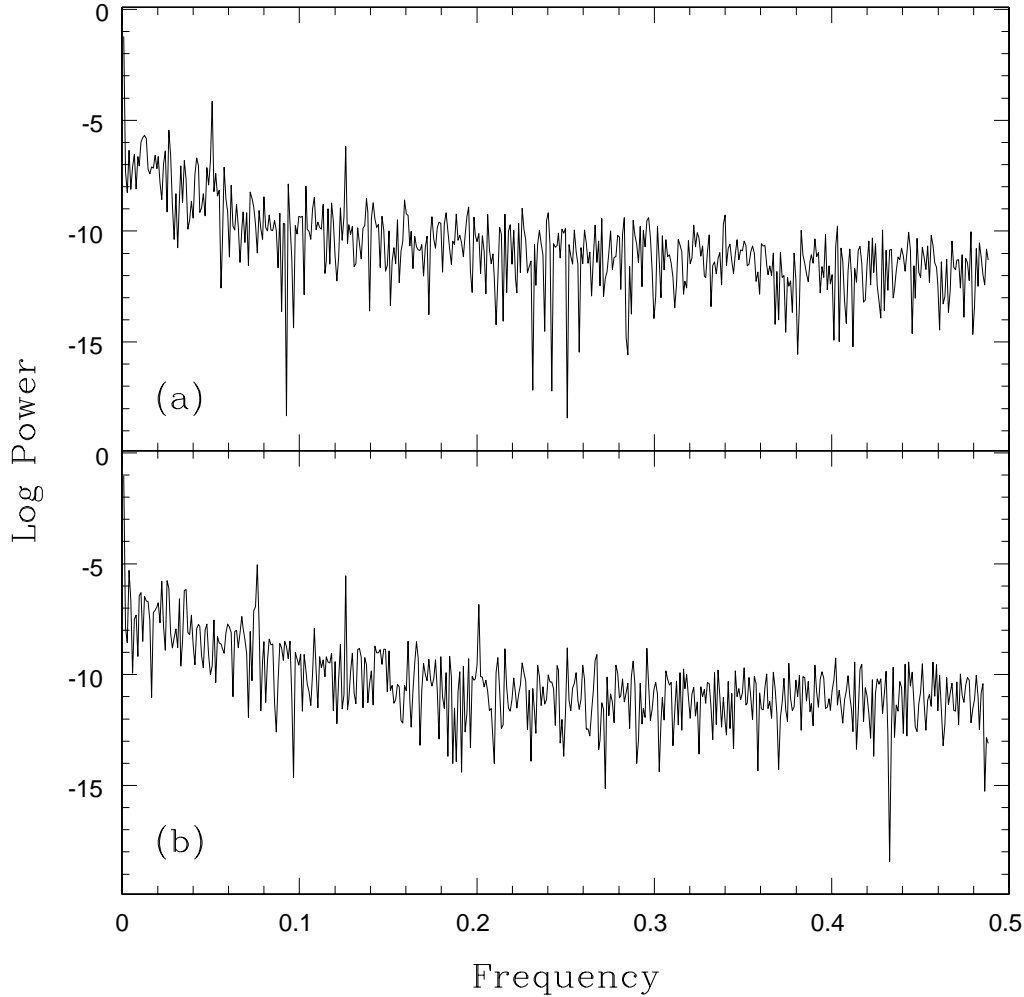


Figure 3. Power spectrum for the bistable system (6) with a composite signal consisting of (a) 2 frequencies $(\nu_1, \nu_2) = (0.125, 0.05)$ and (b) 3 frequencies $(\nu_1, \nu_2, \nu_3) = (0.125, 0.075, 0.2)$, with $Z = 0.16$ and $E = 0.4$ in both cases.

3. The cubic map as a bistable system

Taking $F(t)$ as a composite signal as in equation (9) with $Z = 0.16$ for subthreshold signals of equal amplitude, we drive the system using different combination of frequencies ν_k , with $a = 1.4$ and $b = 0.01$ in the regime of bistable periodic attractors. For convenience, ν_1 is fixed as 0.125 and the other frequencies ν_2, ν_3 etc are varied from 0.02 to 0.4 in steps of 0.005. For each selected combination, the output power spectrum is calculated using the FFT algorithm, for different values of the noise amplitude

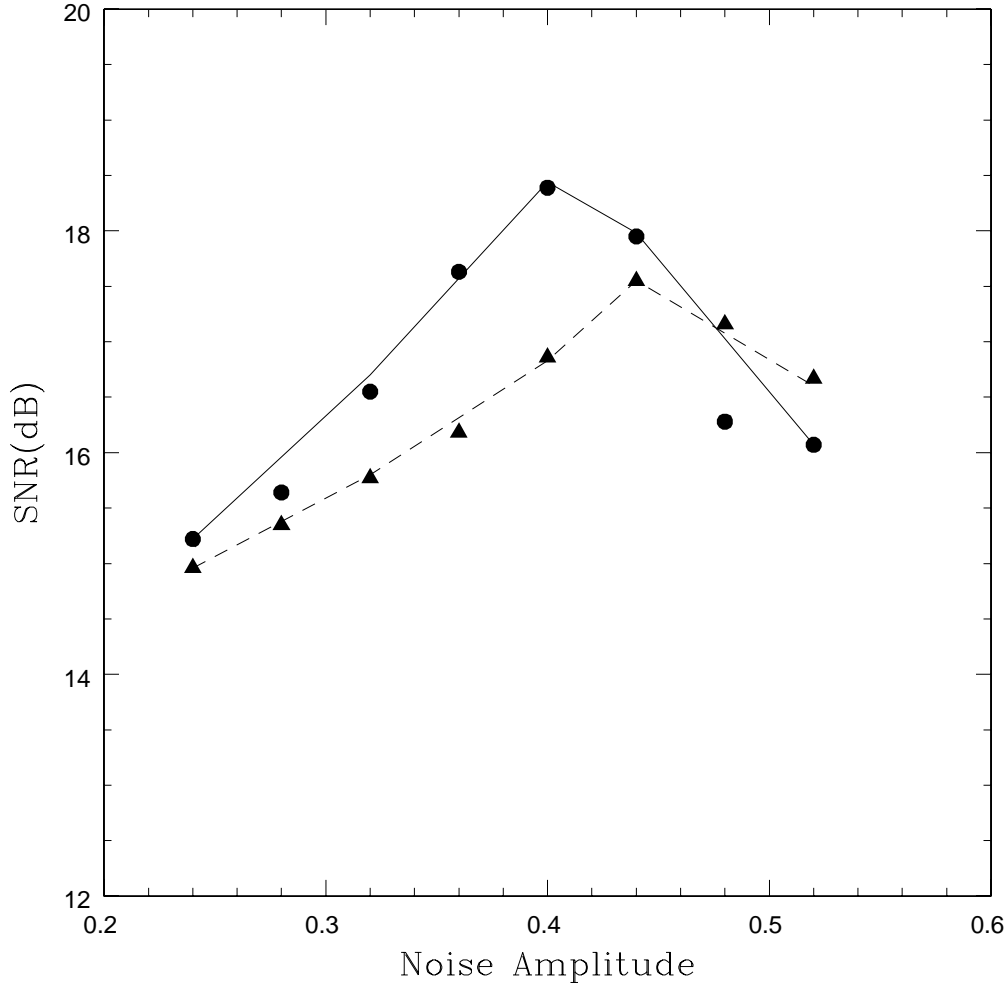


Figure 4. Variation of SNR with noise amplitude for the 2 frequencies shown in Fig.3(a), with the filled circle representing the higher frequency ν_1 .

E. Two typical power spectrums for $n=2$ and $n=3$ with $(\nu_1, \nu_2) = (0.125, 0.05)$ and $(\nu_1, \nu_2, \nu_3) = (0.125, 0.075, 0.2)$ are shown in Fig.3(a) and (b) respectively. To compute the power spectrum, only the inter-well transitions are taken into account and all the intra-well fluctuations are suppressed with a two state filtering. It is clear that only the fundamental frequencies present in the input are enhanced.

We now concentrate on $n=2$ (a combination of 2 frequencies) and compute the two important quantifiers of SR, namely, the SNR and the Residence Time Distribution Function(RTDF). For the frequencies in Fig.3(a), the SNR is computed from the power spectrum using equation (4) for a range of values of E and the results are shown in Fig.4.

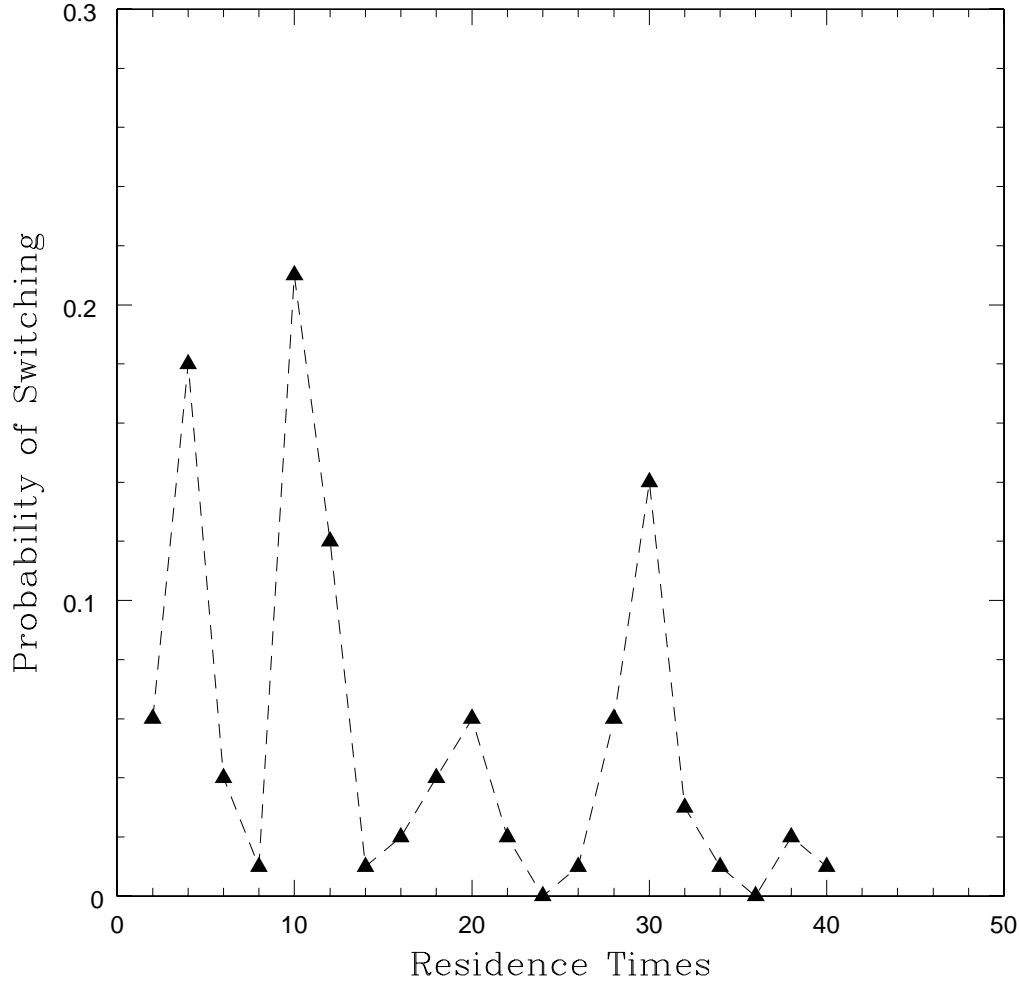


Figure 5. The RTDF for the system (6), with a composite signal consisting of 2 frequencies $\nu_1 = 0.125$ and $\nu_2 = 0.05$. Note that the peaks are synchronised with half the periods corresponding to these frequencies.

The RTDF measures the probability distribution of the average times the system resides in one basin, as a function of different periods. If T is the period of the applied signal, the distribution will have peaks corresponding to times $(2n + 1)T/2$, $n=0,1,2,\dots$. For the system (6) with $(\nu_1, \nu_2) = (0.125, 0.05)$, the results are shown in Fig.5. Note that there are only peaks corresponding to the half integer periods of the two applied frequencies.

The above computations are repeated taking various combinations of frequencies (ν_1, ν_2) , both commensurate and non-commensurate. For a fixed combination of (ν_1, ν_2) , the calculations are also done by changing the signal amplitude Z of one and both

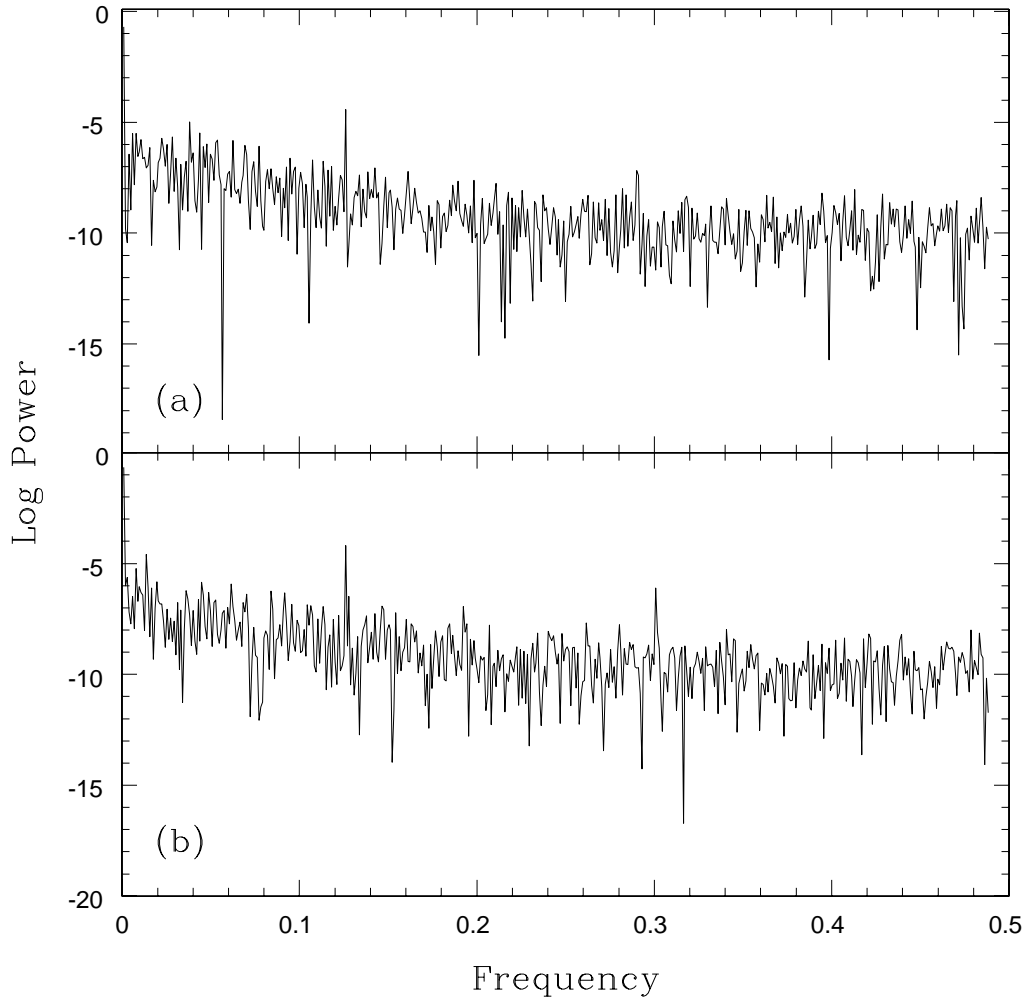


Figure 6. The power spectrum for the bistable system (11), with the signal $F(t)$ consisting of (a) one frequency $\nu_1 = 0.125$ and (b) 2 frequencies $\nu_1 = 0.125$ and $\nu_2 = 0.3$. The parameter values used are $Z = 0.16$, $E = 1.0$, $a = 1.4$ and $b = 0.01$.

signals. Always the results remain qualitatively the same and only the fundamental frequencies present in the input are amplified at the output. If Z becomes very small (< 0.1) compared to the noise level, then the phenomenon of SR disappears altogether and the signal remains undetected in the background noise.

In all the above computations, we used *additive* noise, where the noise has been added to the system externally. But in many natural systems, noise enters through an interaction of the system with the surroundings, that is, through a parameter modulation, rather than a simple addition. Such a *multiplicative* noise occurs in a

variety of physical phenomena [22] and can, in principle, show qualitatively different behavior in the presence of a periodic field [23]. To study its effect on the bistable system, equation (6) is modified as

$$X_{n+1} = b + a(1 + E\zeta(t))X_n - X_n^3 + ZF(t) \quad (11)$$

The noise is added through the parameter a which determines the nature of the bistable attractors. With $a = 1.4$ and $b = 0.01$, the system is now driven by a signal of single frequency $\nu_1 = 0.125$ and a multisignal with 2 frequencies (ν_1, ν_2) , with $Z = 0.16$. The power spectrum for single frequency and multiple frequencies are shown in Fig.6(a) and (b) respectively. The corresponding SNR variation with noise E are shown in Fig.7(a) and (b). Note that the results are qualitatively identical to that of additive noise, but the optimum SNR and the corresponding noise amplitude are comparatively much higher in this case. Thus our numerical results indicate that a bistable system responds only to the fundamental frequencies in a composite signal and not to any mixed modes such as $(\nu_1 - \nu_2)$, in agreement with the LRT. Next we consider the cubic map as a monostable threshold system and show that it behaves differently.

4. The cubic map as a threshold system

As said earlier, the domains of the bistable attractors in the cubic map are clearly separated with the boundary $X = 0$. Hence the cubic map can also be considered as a nondynamical threshold system with a single stable state having a potential barrier. Here the system generates an output *spike* only when the combined effort of the signal and the noise pushes it across the potential barrier ($at X = 0$) in one direction:

$$[E\zeta(t) + ZF(t)] > C_{th} \quad (12)$$

It is then externally reinjected back into the basin. The output thus consists of a series of spikes similar to a random telegraph process. The study of SR in such systems assumes importance in the context of biological applications and in particular the integrate and fire models of neurons where SR has been firmly established[2,24].

The computations are done using equation (6), initially with a single frequency signal, $F(t) = \sin(2\pi\nu_1 t)$. We start from an initial condition in the negative basin and when the output crosses the threshold, $X = 0$, it is reinjected back into the basin by resetting the initial conditions. This is repeated for a sufficiently large number of escapes and the ISIs are calculated. The ISIs are then normalised in terms of the periods T_n and the probability of escape corresponding to each T_n is calculated as the ratio of the number of times T_n occurs to the total number of escapes. The whole procedure is repeated tuning the noise amplitude E . It is found that the ISI is synchronised with the period of the forcing signal for an *optimum* noise amplitude (Fig.8), indicating SR for the frequency ν_1 .

The calculations are now repeated by adding a second signal of frequency ν_2 and amplitude same as that of ν_1 . Again different values of ν_2 in the range 0.02 to 0.4 are used for the calculation. It is then found that apart from the input frequencies ν_1

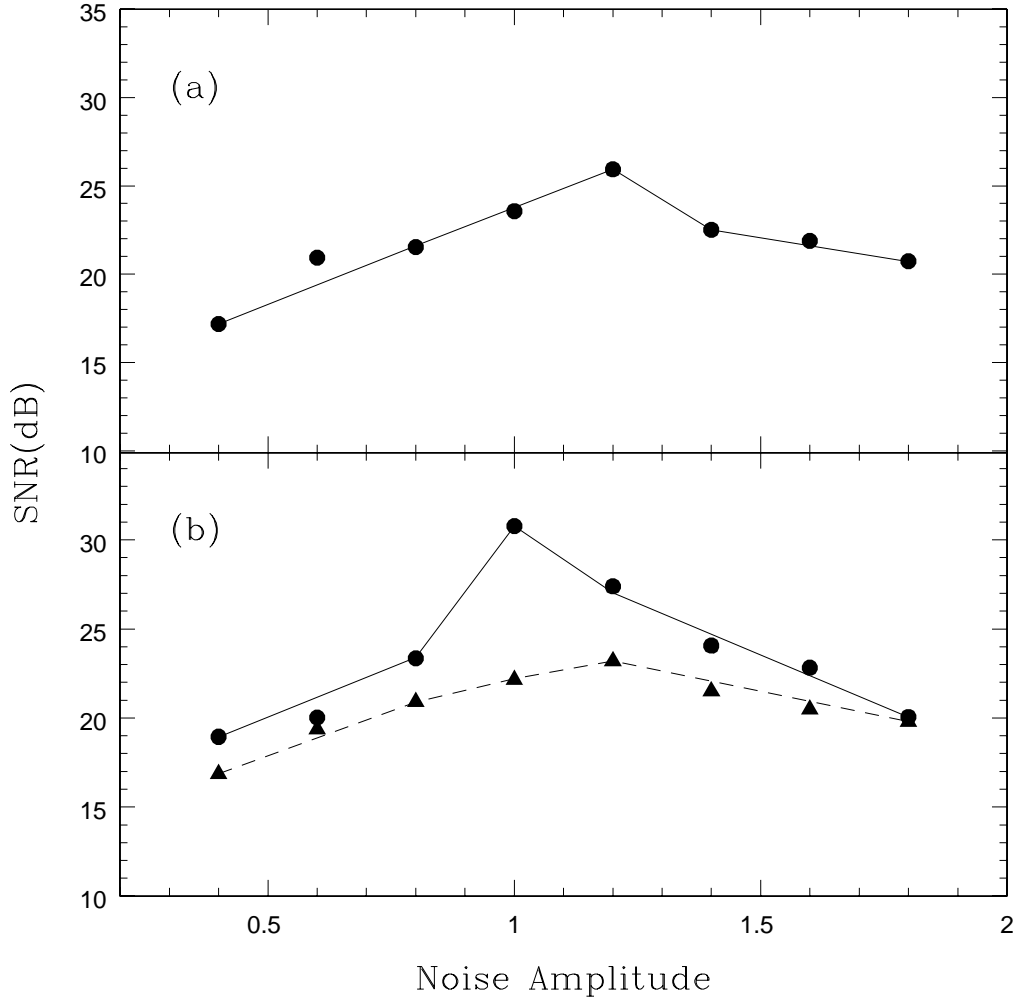


Figure 7. Variation of SNR with noise for the system(11), for (a) single frequency $\nu_1 = 0.125$ and (b) 2 frequencies $\nu_1 = 0.125$ (filled circles) and $\nu_2 = 0.3$. Note that the optimum SNR of ν_1 increases by about 5 dB when a second signal ν_2 is added.

and ν_2 , a third frequency, which is a mixed mode is also enhanced at the output, at a lesser value of the noise amplitude. To make it clear, the amplitude Z of both ν_1 and ν_2 are reduced from 0.16 to 0.08, so that they become too weak to get amplified. The results of computations are shown in Fig.9 and Fig.10, for a combination of input signals $(\nu_1, \nu_2) = (0.125, 0.033)$. Fig.9 represents the probability of escape corresponding to different periods, for the optimum value of noise. It is clear that only a very narrow band of frequencies $d\nu$ around a third frequency ≈ 0.045 (corresponding to the period $T \approx 22s$), are amplified at the output. Note that this frequency is absent in the input

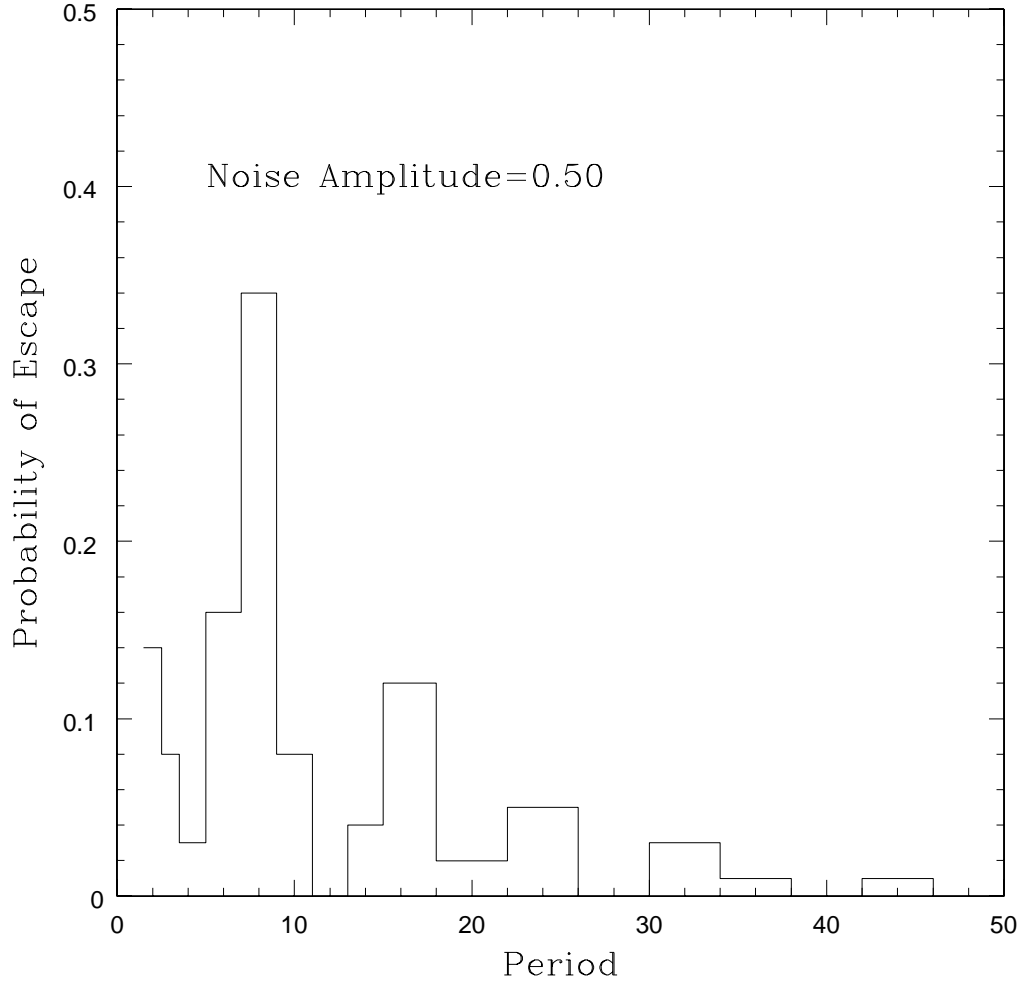


Figure 8. The probability of escape for the system (6) corresponding to different periods, when it is used as a threshold system with the signal $F(t)$ having a single frequency $\nu_1 = 0.125$.

and corresponds to $(\nu_1 - \nu_2)/2$. This is in sharp contrast to the earlier case of a bistable system. The variation of escape probability corresponding to this frequency as a function of noise amplitude is shown in Fig.10.

This result can be understood as follows: When two signals of frequencies ν_1 and ν_2 and equal amplitude Z are superposed, the resulting signal consists of peaks of amplitude $2Z$ repeating with a frequency $(\nu_1 - \nu_2)/2$ in accordance with the linear superposition principle:

$$\sin(2\pi\nu_1 t) + \sin(2\pi\nu_2 t) = 2\sin(2\pi\nu_+ t)\cos(2\pi\nu_- t) \quad (13)$$

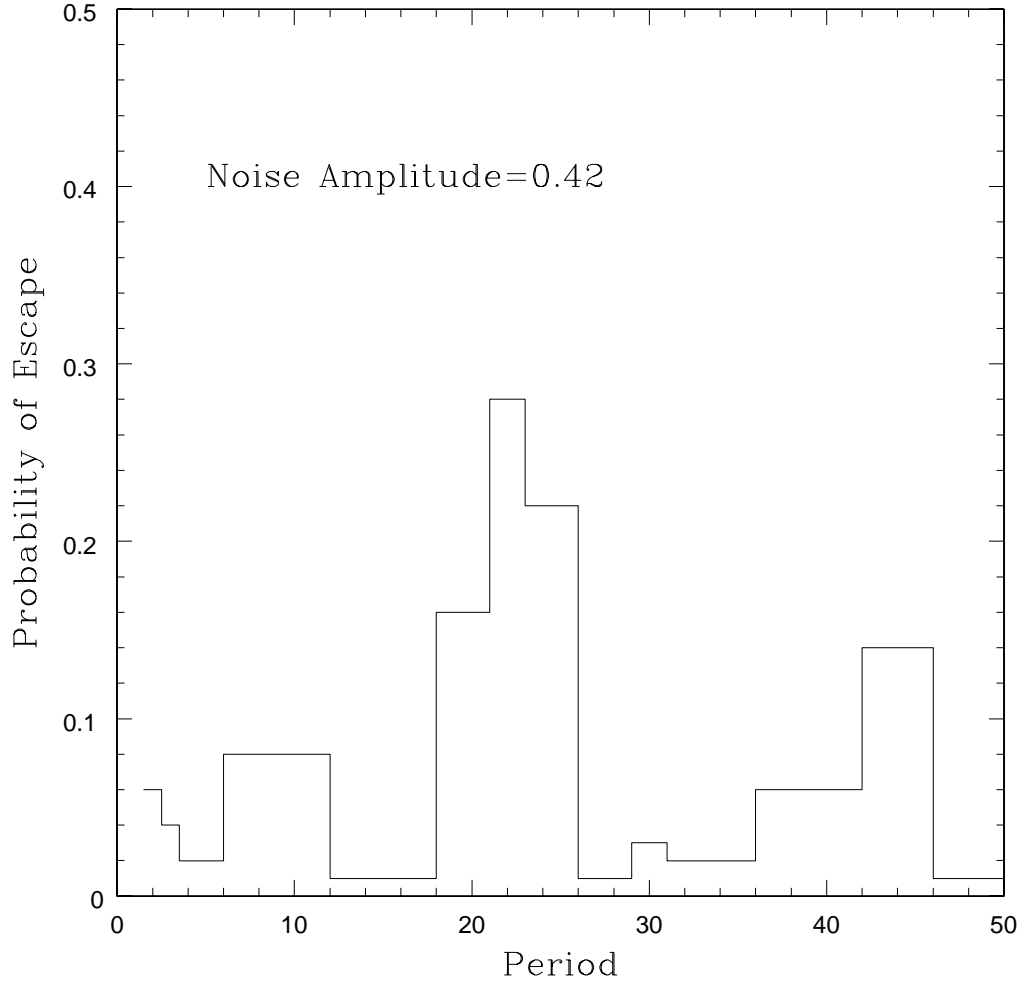


Figure 9. Same as *Fig.8*, but with the signal $F(t)$ comprising of 2 frequencies $\nu_1 = 0.125$ and $\nu_2 = 0.033$ with the individual amplitudes below that required for SR. Note that only a small band of frequencies around $(\nu_1 - \nu_2)/2$ are amplified.

where $\nu_+ = (\nu_1 + \nu_2)/2$ and $\nu_- = (\nu_1 - \nu_2)/2$ as explicitly shown in *Fig.2*. For a threshold system, the probability of escape depends only on the amplitude of the signal which is maximum corresponding to the frequency ν_- . But in the case of a bistable system, the signal is enhanced only if there is a regular shuttling between the wells at the corresponding frequency. This is rather difficult for the frequency ν_- because, its amplitude is modulated by a higher frequency ν_+ . This result has been checked by using different combinations of frequencies (ν_1, ν_2) and also with different amplitudes. It should be mentioned here that this result reveals a fundamental difference between

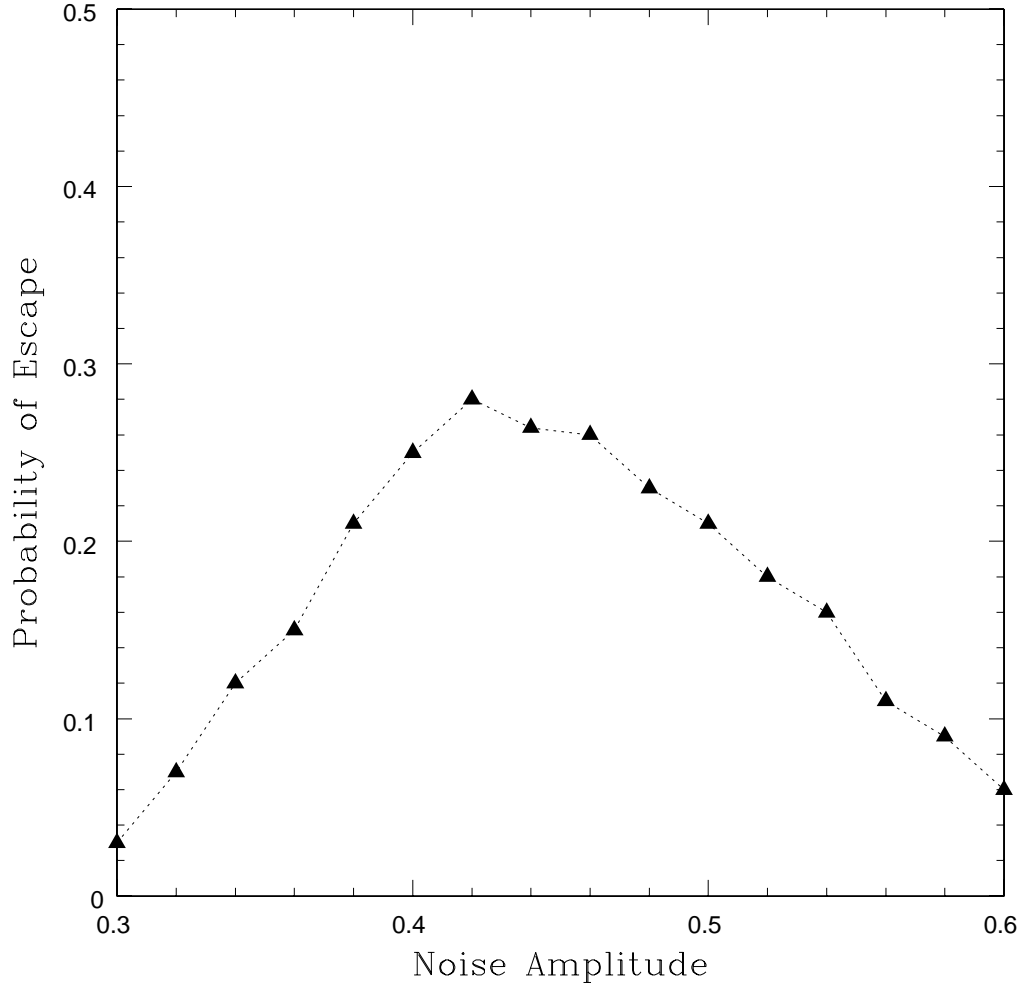


Figure 10. Variation of the probability of escape corresponding to the frequency $(\nu_1 - \nu_2)/2$ for the system (6) as a function of noise amplitude, when it is used as a threshold system with an input signal consisting of 2 frequencies ν_1 and ν_2 .

the two mechanisms of SR and is independent of the model considered here. We have obtained identical results with a fundamentally different model showing SR, namely, a model for Josephson junction and has been discussed elsewhere[15].

5. Results and discussion

In this paper we undertake a detailed numerical study of SR with a bichromatic input signal and gaussian white noise, in both the bistable and threshold set ups.

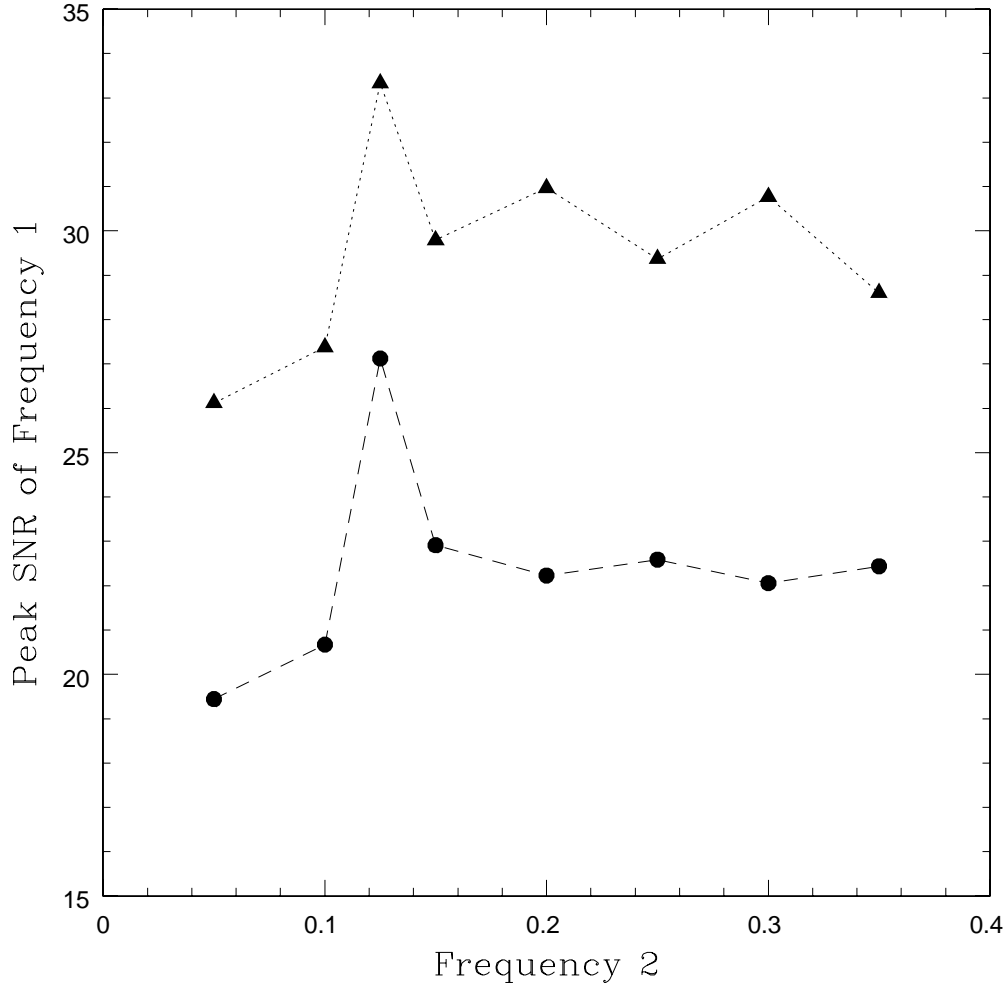


Figure 11. Variation of the optimum value of SNR for the frequency ν_1 as the second frequency ν_2 is varied, for additive noise (filled circles) and multiplicative noise (filled triangles). Note that for $\nu_1 = \nu_2$, the SNR is considerably enhanced in both cases.

We use a simple model for this purpose and both additive and multiplicative noise are used for driving the system in the bistable set up. Our analysis reveals some fundamental differences between the two mechanisms of SR with respect to amplification of multisignal inputs. In particular, we find that, while the bistable set up responds only to the fundamental frequencies present in the input signal, the threshold mechanism enhances a mixed mode also.

An interesting result we have obtained with a bistable system is that the SNR of a signal ν_1 can be improved in general by adding a second signal ν_2 , of higher frequency.

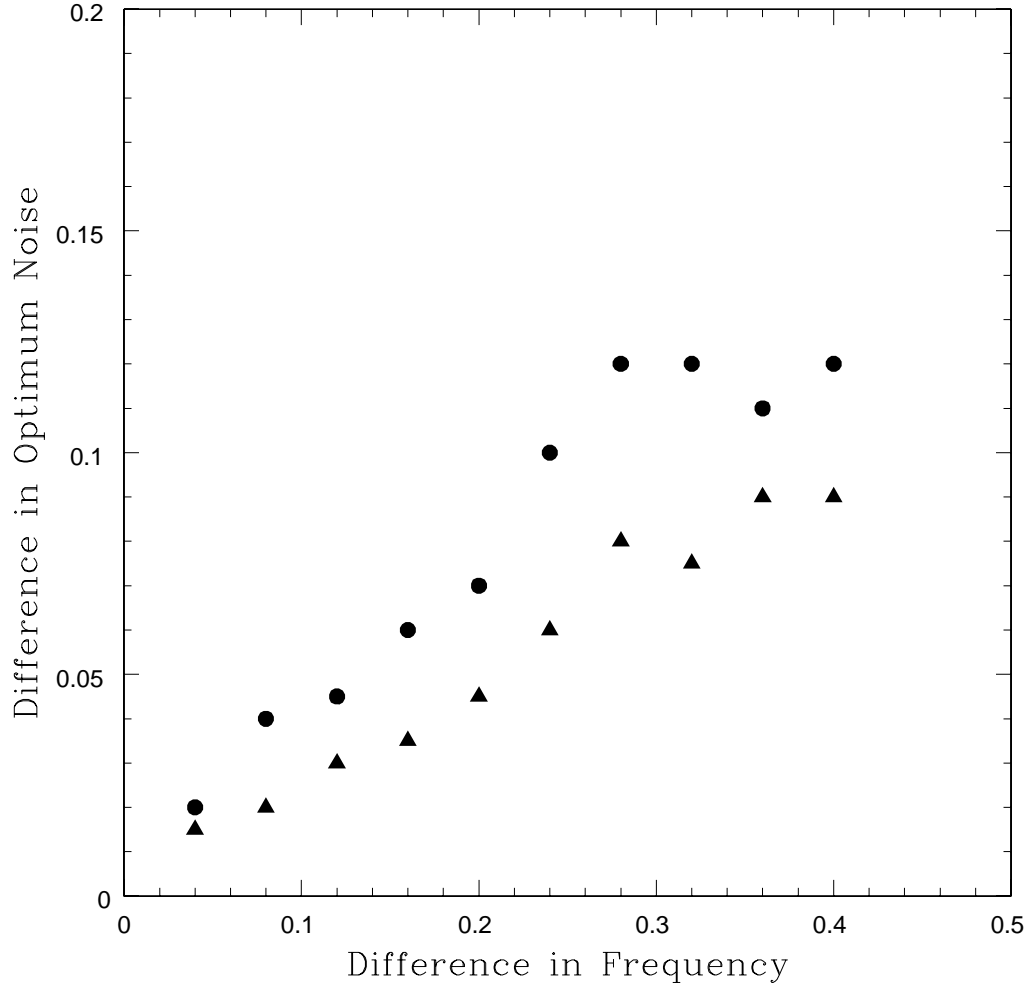


Figure 12. The difference in the noise amplitudes ΔE corresponding to optimum SNRs plotted as a function of the difference in the input frequencies $\Delta\nu$, for a signal consisting of 2 frequencies. Variation is almost similar both for additive noise (filled circles) and multiplicative noise (filled triangles).

This is evident from fig.7 for multiplicative noise. It shows certain co-operative behavior between the two signals. To get a better understanding of the phenomenon, we plot in Fig.11 the optimum SNR value of the first frequency $\nu_1 = 0.125$ for a range of values of ν_2 , for additive as well as multiplicative noise. It is clear that the presence of a second signal of higher frequency enhances the signal detectability of the first one by improving its SNR. Moreover, if the second signal is of the same frequency, the SNR is improved considerably indicating some resonance like phenomenon. Similar results, of using a

high frequency driving for improving the detection of a low frequency signal, have been presented earlier [14,15] under specific dynamical set ups. But the simplicity of our model suggests that the result could be true in general.

Another important result that has emerged from our numerical studies is the possibility of using SR as a *filter* for the detection or selective transmission of the fundamental frequencies in a composite signal using a bistable nonlinear medium and tuning the noise amplitude. For example, it can be seen from Fig.4 and 7 that the noise amplitudes for the optimum SNR for the two frequencies are different. The difference ΔE varies with the difference in the frequencies, $|\nu_1 - \nu_2| (\equiv \Delta\nu)$, as shown in Fig.12 for both types of noise. Note that here the amplitudes of the two signals are equal, Z . The difference ΔE can be made to increase further by tuning the signal amplitudes suitably. Moreover, if the amplitude of one component goes below a minimum threshold value, the SR does not occur and the corresponding component is suppressed in the output. This suggests that SR can, in principle, be used as an effective tool for signal detection/transmission in noisy environments. This can be achieved basically in two ways, either by tuning the noise amplitude or by tuning the amplitude/frequency of the input signals for systems with fixed noise background. A similar idea has been proposed recently [12] in connection with the signal propagation along a one dimensional chain of coupled overdamped oscillators. There it was shown that noise can be used to select the harmonic components propagated with higher efficiency along the chain.

In the threshold mechanism of SR with a composite input signal, a frequency absent in the input signal is enhanced in the system response, a result that is not possible in the context of linear signal processing. This also could have potential practical applications, especially in the study of neuronal mechanism underlying the detection of pitch of complex tones[13,26]. Thus the two mechanisms of SR are different from a practical point of view as well and requires a more detailed analysis.

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